

SC708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Three Level Models

A 3-level HLM model is a logical extension of the 2-level model. Similar model building strategies apply, although now both level 1 and level 2 slopes may vary.

The data for our example are stored in EG1.SAV, EG2.SAV, and EG3.SAV in Chapter 12 under Examples in HLM folder. These data consist of 7230 observations collected on 1721 children from 60 schools, beginning at the end of grade one and followed up annually thereafter until grade six. The students are the level-2 units, the schools are the level-3 units. The outcome variable of interest is the result of a math test, represented by the variable MATH. On level 1, the information available includes the year of the study minus 3.5 (denoted by the variable YEAR), the grade level minus 1.0 at each testing occasion (denoted by GRADE) and an indicator that a child is retained in grade for a particular year (denoted by the variable RETAINED, which assumes a value of 1 if retained, 0 otherwise). At level-2, the following variables are available: GENDER, denoting the gender of the child (1=female), BLACK, denoting whether the child is Black, and HISPANIC, denoting whether the child is Hispanic. At level-3, the following variables are available: SIZE, measuring the size of school, LOWINC indicating the percentage of low-income students in the school, and MOBILITY indicating the level of residential mobility (%).

The creation of the MDM file for a 3-level model is very similar; however, make sure you sort your data by both level 3 and level 2 ID.

Let's start by modeling the linear growth trajectory unconditionally, with random effects at levels 2 and 3:

```

Level-1 Model
  Y = P0 + P1*(YEAR) + E
Level-2 Model
  P0 = B00 + R0
  P1 = B10 + R1
Level-3 Model
  B00 = G000 + U00
  B10 = G100 + U10

Sigma_squared =          0.30148

Standard Error of Sigma_squared =          0.00660

Tau(pi)
INTRCPT1,P0          0.64049          0.04676
YEAR,P1              0.04676          0.01122

Tau(pi) (as correlations)
INTRCPT1,P0          1.000          0.551
YEAR,P1              0.551          1.000

Standard Errors of Tau(pi)
INTRCPT1,P0          0.02515          0.00499
YEAR,P1              0.00499          0.00196

```

Random level-1 coefficient	Reliability estimate
INTRCPT1, P0	0.839
YEAR, P1	0.190

Tau(beta)

INTRCPT1	YEAR
INTRCPT2,B00	INTRCPT2,B10
0.16531	0.01705
0.01705	0.01102

Tau(beta) (as correlations)

INTRCPT1/INTRCPT2,B00	1.000	0.399
YEAR/INTRCPT2,B10	0.399	1.000

Standard Errors of Tau(beta)

INTRCPT1	YEAR
INTRCPT2,B00	INTRCPT2,B10
0.03641	0.00720
0.00720	0.00252

Random level-2 coefficient	Reliability estimate
INTRCPT1/INTRCPT2, B00	0.821
YEAR/INTRCPT2, B10	0.786

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.779309	0.057829	-13.476	59	0.000
For YEAR slope, P1					
For INTRCPT2, B10					
INTRCPT3, G100	0.763029	0.015263	49.993	59	0.000

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.779309	0.057830	-13.476	59	0.000
For YEAR slope, P1					
For INTRCPT2, B10					
INTRCPT3, G100	0.763029	0.015260	50.000	59	0.000

Final estimation of level-1 and level-2 variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, R0	0.80030	0.64049	1661	6046.38093	0.000
YEAR slope, R1	0.10595	0.01122	1661	2083.62359	0.000
level-1, E	0.54907	0.30148			

Final estimation of level-3 variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1/INTRCPT2, U00	0.40658	0.16531	59	488.34492	0.000
YEAR/INTRCPT2, U10	0.10498	0.01102	59	377.40852	0.000

Statistics for current covariance components model

Deviance = 16326.231292
 Number of estimated parameters = 9

Now we can add some explanatory variables on level 2.

Level-1 Model

$$Y = P0 + P1*(YEAR) + E$$

Level-2 Model

$$P0 = B00 + B01*(BLACK) + B02*(HISPANIC) + R0$$

$$P1 = B10 + B11*(BLACK) + B12*(HISPANIC) + R1$$

Level-3 Model

$$B00 = G000 + U00$$

$$B01 = G010 + U01$$

$$B02 = G020 + U02$$

$$B10 = G100 + U10$$

$$B11 = G110 + U11$$

$$B12 = G120 + U12$$

Sigma_squared = 0.30154

Standard Error of Sigma_squared = 0.00660

Tau(pi)

INTRCPT1,P0	0.62029	0.04599
YEAR,P1	0.04599	0.01078

Tau(pi) (as correlations)

INTRCPT1,P0	1.000	0.563
YEAR,P1	0.563	1.000

Standard Errors of Tau(pi)

INTRCPT1,P0	0.02471	0.00495
YEAR,P1	0.00495	0.00197

 Random level-1 coefficient Reliability estimate

INTRCPT1, P0	0.835
YEAR, P1	0.184

Tau(beta)

INTRCPT1	INTRCPT1	INTRCPT1	YEAR	YEAR	YEAR
INTRCPT2,B00	BLACK,B01	HISPANIC,B02	INTRCPT2,B10	BLACK,B11	HISPANIC,B12
0.09278	0.00753	-0.01335	0.01619	-0.01040	-0.00495
0.00753	0.03144	0.01699	-0.00666	0.00706	0.01114
-0.01335	0.01699	0.01455	-0.00793	0.00527	0.00713
0.01619	-0.00666	-0.00793	0.01123	-0.00316	-0.00361
-0.01040	0.00706	0.00527	-0.00316	0.00423	0.00420
-0.00495	0.01114	0.00713	-0.00361	0.00420	0.00518

Tau(beta) (as correlations)

INTRCPT1/INTRCPT2,B00	1.000	0.139	-0.363	0.502	-0.525	-0.226
INTRCPT1/ BLACK,B01	0.139	1.000	0.794	-0.354	0.612	0.873
INTRCPT1/HISPANIC,B02	-0.363	0.794	1.000	-0.620	0.672	0.821
YEAR/INTRCPT2,B10	0.502	-0.354	-0.620	1.000	-0.459	-0.473
YEAR/ BLACK,B11	-0.525	0.612	0.672	-0.459	1.000	0.897
YEAR/HISPANIC,B12	-0.226	0.873	0.821	-0.473	0.897	1.000

Standard Errors of Tau(beta)

INTRCPT1	INTRCPT1	INTRCPT1	YEAR	YEAR	YEAR
INTRCPT2,B00	BLACK,B01	HISPANIC,B02	INTRCPT2,B10	BLACK,B11	HISPANIC,B12
0.04403	0.03696	0.04083	0.01079	0.01152	0.01232
0.03696	0.05652	0.04714	0.01221	0.01281	0.01503
0.04083	0.04714	0.05788	0.01250	0.01392	0.01384
0.01079	0.01221	0.01250	0.00457	0.00393	0.00431
0.01152	0.01281	0.01392	0.00393	0.00533	0.00475
0.01232	0.01503	0.01384	0.00431	0.00475	0.00609

Random level-2 coefficient	Reliability estimate
INTRCPT1/INTRCPT2, B00	0.448
INTRCPT1/ BLACK, B01	0.097
INTRCPT1/HISPANIC, B02	0.048
YEAR/INTRCPT2, B10	0.532
YEAR/ BLACK, B11	0.136
YEAR/HISPANIC, B12	0.170

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.345330	0.072627	-4.755	59	0.000
For BLACK, B01					
INTRCPT3, G010	-0.603308	0.079520	-7.587	59	0.000
For HISPANIC, B02					
INTRCPT3, G020	-0.402046	0.086434	-4.651	59	0.000
For YEAR slope, P1					
For INTRCPT2, B10					
INTRCPT3, G100	0.786289	0.023389	33.618	59	0.000
For BLACK, B11					
INTRCPT3, G110	-0.051662	0.024618	-2.099	59	0.040
For HISPANIC, B12					
INTRCPT3, G120	0.033075	0.027829	1.189	59	0.240

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.345330	0.070148	-4.923	59	0.000
For BLACK, B01					
INTRCPT3, G010	-0.603308	0.074322	-8.117	59	0.000
For HISPANIC, B02					
INTRCPT3, G020	-0.402046	0.079362	-5.066	59	0.000

```

For      YEAR slope, P1
For INTRCPT2, B10
  INTRCPT3, G100      0.786289      0.023261      33.803      59      0.000
For      BLACK, B11
  INTRCPT3, G110     -0.051662      0.024064      -2.147      59      0.036
For HISPANIC, B12
  INTRCPT3, G120      0.033075      0.026437      1.251      59      0.216
-----

```

Final estimation of level-1 and level-2 variance components:

```

-----
Random Effect          Standard      Variance      df      Chi-square      P-value
                        Deviation    Component
-----
INTRCPT1,      R0      0.78759      0.62029      1541      6780.61188      0.000
  YEAR slope, R1      0.10380      0.01078      1541      2082.57394      0.000
  level-1,      E      0.54912      0.30154
-----

```

Final estimation of level-3 variance components:

```

-----
Random Effect          Standard      Variance      df      Chi-square      P-value
                        Deviation    Component
-----
INTRCPT1/INTRCPT2, U00      0.30460      0.09278      24      71.58653      0.000
INTRCPT1/  BLACK, U01      0.17732      0.03144      24      32.61766      0.112
INTRCPT1/HISPANIC, U02      0.12063      0.01455      24      32.88948      0.106
  YEAR/INTRCPT2, U10      0.10598      0.01123      24      64.79784      0.000
  YEAR/  BLACK, U11      0.06503      0.00423      24      22.73325      >.500
  YEAR/HISPANIC, U12      0.07200      0.00518      24      21.67923      >.500
-----

```

Finally, we can add some level 3 predictors as well. Since there is no variation in the effects of race/ethnicity across schools, we will fix the effects of Black and Hispanic.

Level-1 Model

$$Y = P_0 + P_1 * (\text{YEAR}) + E$$

Level-2 Model

$$P_0 = B_{00} + B_{01} * (\text{BLACK}) + B_{02} * (\text{HISPANIC}) + R_0$$

$$P_1 = B_{10} + B_{11} * (\text{BLACK}) + B_{12} * (\text{HISPANIC}) + R_1$$

Level-3 Model

$$B_{00} = G_{000} + G_{001} (\text{SIZE}) + G_{002} (\text{LOWINC}) + G_{003} (\text{MOBILITY}) + U_{00}$$

$$B_{01} = G_{010}$$

$$B_{02} = G_{020}$$

$$B_{10} = G_{100} + G_{101} (\text{SIZE}) + G_{102} (\text{LOWINC}) + G_{103} (\text{MOBILITY}) + U_{10}$$

$$B_{11} = G_{110}$$

$$B_{12} = G_{120}$$

$$\text{Sigma_squared} = 0.30160$$

$$\text{Standard Error of Sigma_squared} = 0.00660$$

Tau(pi)

```

INTRCPT1, P0      0.62162      0.04651
  YEAR, P1      0.04651      0.01105

```

Tau(pi) (as correlations)

```

INTRCPT1, P0      1.000      0.561
  YEAR, P1      0.561      1.000

```

Standard Errors of Tau(pi)

```

INTRCPT1, P0      0.02449      0.00491

```

YEAR, P1 0.00491 0.00196

Random level-1 coefficient	Reliability estimate
INTRCPT1, P0	0.835
YEAR, P1	0.187

Tau(beta)
 INTRCPT1 YEAR
 INTRCPT2,B00 INTRCPT2,B10
 0.06128 -0.00071
 -0.00071 0.00772

Tau(beta) (as correlations)
 INTRCPT1/INTRCPT2,B00 1.000 -0.033
 YEAR/INTRCPT2,B10 -0.033 1.000

Standard Errors of Tau(beta)
 INTRCPT1 YEAR
 INTRCPT2,B00 INTRCPT2,B10
 0.01668 0.00397
 0.00397 0.00189

Random level-2 coefficient	Reliability estimate
INTRCPT1/INTRCPT2, B00	0.654
YEAR/INTRCPT2, B10	0.729

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.409246	0.069468	-5.891	56	0.000
SIZE, G001	-0.000040	0.000132	-0.301	56	0.764
LOWINC, G002	-0.004590	0.001794	-2.559	56	0.014
MOBILITY, G003	-0.011756	0.003428	-3.430	56	0.001
For BLACK, B01					
INTRCPT3, G010	-0.517660	0.076949	-6.727	1718	0.000
For HISPANIC, B02					
INTRCPT3, G020	-0.322289	0.085851	-3.754	1718	0.000
For YEAR slope, P1					
For INTRCPT2, B10					
INTRCPT3, G100	0.774828	0.020869	37.129	56	0.000
SIZE, G101	-0.000053	0.000045	-1.184	56	0.242
LOWINC, G102	-0.001084	0.000595	-1.823	56	0.073
MOBILITY, G103	-0.000577	0.001174	-0.492	56	0.624
For BLACK, B11					
INTRCPT3, G110	-0.032248	0.022438	-1.437	1718	0.151
For HISPANIC, B12					
INTRCPT3, G120	0.044658	0.024698	1.808	1718	0.070

Final estimation of fixed effects
 (with robust standard errors)

Standard Approx.

Fixed Effect	Coefficient	Error	T-ratio	d.f.	P-value
For INTRCPT1, P0					
For INTRCPT2, B00					
INTRCPT3, G000	-0.409246	0.068171	-6.003	56	0.000
SIZE, G001	-0.000040	0.000166	-0.239	56	0.812
LOWINC, G002	-0.004590	0.001890	-2.429	56	0.019
MOBILITY, G003	-0.011756	0.004375	-2.687	56	0.010
For BLACK, B01					
INTRCPT3, G010	-0.517660	0.075851	-6.825	1718	0.000
For HISPANIC, B02					
INTRCPT3, G020	-0.322289	0.080945	-3.982	1718	0.000
For YEAR slope, P1					
For INTRCPT2, B10					
INTRCPT3, G100	0.774828	0.020791	37.268	56	0.000
SIZE, G101	-0.000053	0.000047	-1.148	56	0.256
LOWINC, G102	-0.001084	0.000536	-2.022	56	0.048
MOBILITY, G103	-0.000577	0.001020	-0.566	56	0.573
For BLACK, B11					
INTRCPT3, G110	-0.032248	0.022206	-1.452	1718	0.147
For HISPANIC, B12					
INTRCPT3, G120	0.044658	0.023342	1.913	1718	0.055

Final estimation of level-1 and level-2 variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1, R0	0.78843	0.62162	1659	8734.26910	0.000
YEAR slope, R1	0.10514	0.01105	1659	2098.08638	0.000
level-1, E	0.54918	0.30160			

Final estimation of level-3 variance components:

Random Effect	Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1/INTRCPT2, U00	0.24754	0.06128	56	224.59986	0.000
YEAR/INTRCPT2, U10	0.08784	0.00772	56	272.69172	0.000

Note: HMLM2 module of HLM software allows estimation of growth models with complex covariance structures for 3-level models.