

SC708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Testing Hypotheses about Simple Slopes for Interactions

SES as the main variable and SECTOR as a moderator

Focusing on SES as main variable and SECTOR as a moderator, we can calculate the significance of the effect of SES in Catholic schools (when SECTOR=1):

Final Results - Iteration 242

Iterations stopped due to small change in likelihood function

$\sigma^2 = 36.36622$

τ

INTRCPT1, β_0	4.27594	-0.99430	0.62032
FEMALE, β_1	-0.99430	0.89361	-0.15189
SES, β_2	0.62032	-0.15189	0.12308

τ (as correlations)

INTRCPT1, β_0	1.000	-0.509	0.855
FEMALE, β_1	-0.509	1.000	-0.458
SES, β_2	0.855	-0.458	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1, β_0	0.650
FEMALE, β_1	0.196
SES, β_2	0.060

Note: The reliability estimates reported above are based on only 123 of 160 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

The value of the log-likelihood function at iteration 242 = -2.325021E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, Y_{00}	12.264554	0.407791	30.076	156	<0.001
SECTOR, Y_{01}	2.370436	0.443345	5.347	156	<0.001
SIZED1, Y_{02}	-1.230907	0.577382	-2.132	156	0.035
SIZED3, Y_{03}	0.523767	0.475612	1.101	156	0.272
For FEMALE slope, β_1					
INTRCPT2, Y_{10}	-0.992369	0.360317	-2.754	156	0.007
SECTOR, Y_{11}	-0.174713	0.413095	-0.423	156	0.673
SIZED1, Y_{12}	0.711859	0.527660	1.349	156	0.179
SIZED3, Y_{13}	-0.598786	0.431117	-1.389	156	0.167
For SES slope, β_2					
INTRCPT2, Y_{20}	2.815012	0.223272	12.608	156	<0.001
SECTOR, Y_{21}	-1.284821	0.239249	-5.370	156	<0.001
SIZED1, Y_{22}	0.254340	0.295899	0.860	156	0.391
SIZED3, Y_{23}	0.125232	0.259568	0.482	156	0.630

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, γ_{00}	12.264554	0.0000
SECTOR, γ_{01}	2.370436	0.0000
SIZED1, γ_{02}	-1.230907	0.0000
SIZED3, γ_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, γ_{10}	-0.992369	0.0000
SECTOR, γ_{11}	-0.174713	0.0000
SIZED1, γ_{12}	0.711859	0.0000
SIZED3, γ_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, γ_{20}	2.815012	1.0000
SECTOR, γ_{21}	-1.284821	1.0000
SIZED1, γ_{22}	0.254340	0.0000
SIZED3, γ_{23}	0.125232	0.0000
Estimate		1.5302
Standard error of estimate		0.1971

χ^2 statistic = 60.302971
Degrees of freedom = 1
p-value = <0.001

**Final estimation of fixed effects
(with robust standard errors)**

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.264554	0.406861	30.144	156	<0.001
SECTOR, γ_{01}	2.370436	0.429390	5.520	156	<0.001
SIZED1, γ_{02}	-1.230907	0.580527	-2.120	156	0.036
SIZED3, γ_{03}	0.523767	0.452241	1.158	156	0.249
For FEMALE slope, β_1					
INTRCPT2, γ_{10}	-0.992369	0.371180	-2.674	156	0.008
SECTOR, γ_{11}	-0.174713	0.417936	-0.418	156	0.676
SIZED1, γ_{12}	0.711859	0.505528	1.408	156	0.161
SIZED3, γ_{13}	-0.598786	0.422319	-1.418	156	0.158
For SES slope, β_2					
INTRCPT2, γ_{20}	2.815012	0.209242	13.453	156	<0.001
SECTOR, γ_{21}	-1.284821	0.225575	-5.696	156	<0.001
SIZED1, γ_{22}	0.254340	0.286061	0.889	156	0.375
SIZED3, γ_{23}	0.125232	0.236842	0.529	156	0.598

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, γ_{00}	12.264554	0.0000
SECTOR, γ_{01}	2.370436	0.0000
SIZED1, γ_{02}	-1.230907	0.0000
SIZED3, γ_{03}	0.523767	0.0000
For FEMALE slope, β_1		

INTRCPT2, γ_{10}	-0.992369	0.0000
SECTOR, γ_{11}	-0.174713	0.0000
SIZED1, γ_{12}	0.711859	0.0000
SIZED3, γ_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, γ_{20}	2.815012	1.0000
SECTOR, γ_{21}	-1.284821	1.0000
SIZED1, γ_{22}	0.254340	0.0000
SIZED3, γ_{23}	0.125232	0.0000
Estimate		1.5302
Standard error of estimate		0.1882

χ^2 statistic = 66.130820
 Degrees of freedom = 1
 p-value = <0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	2.06783	4.27594	119	348.85471	<0.001
FEMALE slope, u_1	0.94531	0.89361	119	139.73919	0.094
SES slope, u_2	0.35083	0.12308	119	129.90176	0.233
level-1, r	6.03044	36.36622			

Note: The chi-square statistics reported above are based on only 123 of 160 units that had sufficient data for computation. Fixed effects and variance components are based on all the data.

Statistics for current covariance components model

Deviance = 46500.428022
 Number of estimated parameters = 7

Note that I used HLM 7 and it conducts hypothesis testing twice – one based on regular standard errors, and another time based on robust ones. Based on robust ones, then, we see:

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, γ_{00}	12.264554	0.0000
SECTOR, γ_{01}	2.370436	0.0000
SIZED1, γ_{02}	-1.230907	0.0000
SIZED3, γ_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, γ_{10}	-0.992369	0.0000
SECTOR, γ_{11}	-0.174713	0.0000
SIZED1, γ_{12}	0.711859	0.0000
SIZED3, γ_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, γ_{20}	2.815012	1.0000

SECTOR, Y_{21}	-1.284821	1.0000
SIZED1, Y_{22}	0.254340	0.0000
SIZED3, Y_{23}	0.125232	0.0000
Estimate		1.5302
Standard error of estimate		0.1882

χ^2 statistic = 66.130820
 Degrees of freedom = 1
 p-value = <0.001

Thus, the coefficient reflecting the effect of SES when SECTOR=1 is 1.5302, and its significance is reflected by p -value = <0.001.

SECTOR as the main variable and SES as a moderator

Now let's focus on SECTOR as the main variable and calculate and test its effects when SES = -1 and SES = 2. We can do that by specifying both hypothesis 1 and hypothesis 2. Here are the relevant pieces of the output:

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, Y_{00}	12.264554	0.407791	30.076	156	<0.001
SECTOR, Y_{01}	2.370436	0.443345	5.347	156	<0.001
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SIZED3, Y_{13}	-0.598786	0.431117	-1.389	156	0.167
For SES slope, β_2					
INTRCPT2, Y_{20}	2.815012	0.223272	12.608	156	<0.001
SECTOR, Y_{21}	-1.284821	0.239249	-5.370	156	<0.001
SIZED1, Y_{22}	0.254340	0.295899	0.860	156	0.391
SIZED3, Y_{23}	0.125232	0.259568	0.482	156	0.630

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, Y_{00}	12.264554	0.0000
SECTOR, Y_{01}	2.370436	1.0000
SIZED1, Y_{02}	-1.230907	0.0000
SIZED3, Y_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, Y_{10}	-0.992369	0.0000
SECTOR, Y_{11}	-0.174713	0.0000
SIZED1, Y_{12}	0.711859	0.0000
SIZED3, Y_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, Y_{20}	2.815012	0.0000

SECTOR, Y_{21}	-1.284821	-1.0000
SIZED1, Y_{22}	0.254340	0.0000
SIZED3, Y_{23}	0.125232	0.0000
Estimate		3.6553
Standard error of estimate		0.4647

χ^2 statistic = 61.881857
Degrees of freedom = 1
p-value = <0.001

Results of General Linear Hypothesis Testing - Test 2

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, Y_{00}	12.264554	0.0000
SECTOR, Y_{01}	2.370436	1.0000
SIZED1, Y_{02}	-1.230907	0.0000
SIZED3, Y_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, Y_{10}	-0.992369	0.0000
SECTOR, Y_{11}	-0.174713	0.0000
SIZED1, Y_{12}	0.711859	0.0000
SIZED3, Y_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, Y_{20}	2.815012	0.0000
SECTOR, Y_{21}	-1.284821	2.0000
SIZED1, Y_{22}	0.254340	0.0000
SIZED3, Y_{23}	0.125232	0.0000
Estimate		-0.1992
Standard error of estimate		0.7080

χ^2 statistic = 0.079161
Degrees of freedom = 1
p-value = >.500

Final estimation of fixed effects (with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, Y_{00}	12.264554	0.406861	30.144	156	<0.001
SECTOR, Y_{01}	2.370436	0.429390	5.520	156	<0.001
SIZED1, Y_{02}	-1.230907	0.580527	-2.120	156	0.036
SIZED3, Y_{03}	0.523767	0.452241	1.158	156	0.249
For FEMALE slope, β_1					
INTRCPT2, Y_{10}	-0.992369	0.371180	-2.674	156	0.008
SECTOR, Y_{11}	-0.174713	0.417936	-0.418	156	0.676
SIZED1, Y_{12}	0.711859	0.505528	1.408	156	0.161
SIZED3, Y_{13}	-0.598786	0.422319	-1.418	156	0.158
For SES slope, β_2					
INTRCPT2, Y_{20}	2.815012	0.209242	13.453	156	<0.001
SECTOR, Y_{21}	-1.284821	0.225575	-5.696	156	<0.001
SIZED1, Y_{22}	0.254340	0.286061	0.889	156	0.375
SIZED3, Y_{23}	0.125232	0.236842	0.529	156	0.598

Results of General Linear Hypothesis Testing - Test 1

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, Y_{00}	12.264554	0.0000
SECTOR, Y_{01}	2.370436	1.0000
SIZED1, Y_{02}	-1.230907	0.0000
SIZED3, Y_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, Y_{10}	-0.992369	0.0000
SECTOR, Y_{11}	-0.174713	0.0000
SIZED1, Y_{12}	0.711859	0.0000
SIZED3, Y_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, Y_{20}	2.815012	0.0000
SECTOR, Y_{21}	-1.284821	-1.0000
SIZED1, Y_{22}	0.254340	0.0000
SIZED3, Y_{23}	0.125232	0.0000
Estimate		3.6553
Standard error of estimate		0.4661

χ^2 statistic = 61.489379
 Degrees of freedom = 1
 p-value = <0.001

Results of General Linear Hypothesis Testing - Test 2

	Coefficients	Contrast
For INTRCPT1, β_0		
INTRCPT2, Y_{00}	12.264554	0.0000
SECTOR, Y_{01}	2.370436	1.0000
SIZED1, Y_{02}	-1.230907	0.0000
SIZED3, Y_{03}	0.523767	0.0000
For FEMALE slope, β_1		
INTRCPT2, Y_{10}	-0.992369	0.0000
SECTOR, Y_{11}	-0.174713	0.0000
SIZED1, Y_{12}	0.711859	0.0000
SIZED3, Y_{13}	-0.598786	0.0000
For SES slope, β_2		
INTRCPT2, Y_{20}	2.815012	0.0000
SECTOR, Y_{21}	-1.284821	2.0000
SIZED1, Y_{22}	0.254340	0.0000
SIZED3, Y_{23}	0.125232	0.0000
Estimate		-0.1992
Standard error of estimate		0.6510

χ^2 statistic = 0.093622
 Degrees of freedom = 1
 p-value = >.500

Based on this last one (robust SE) set of results, the effect of SECTOR when SES=-1 is 3.6553 and statistically significant, and the effect of SECTOR when SES=2 is -0.1992 and is not statistically significant.