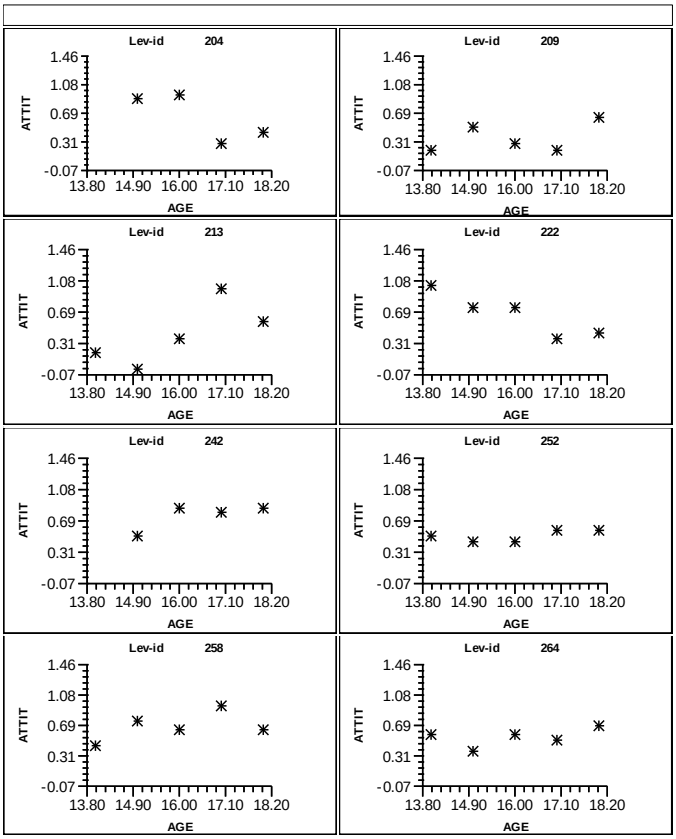
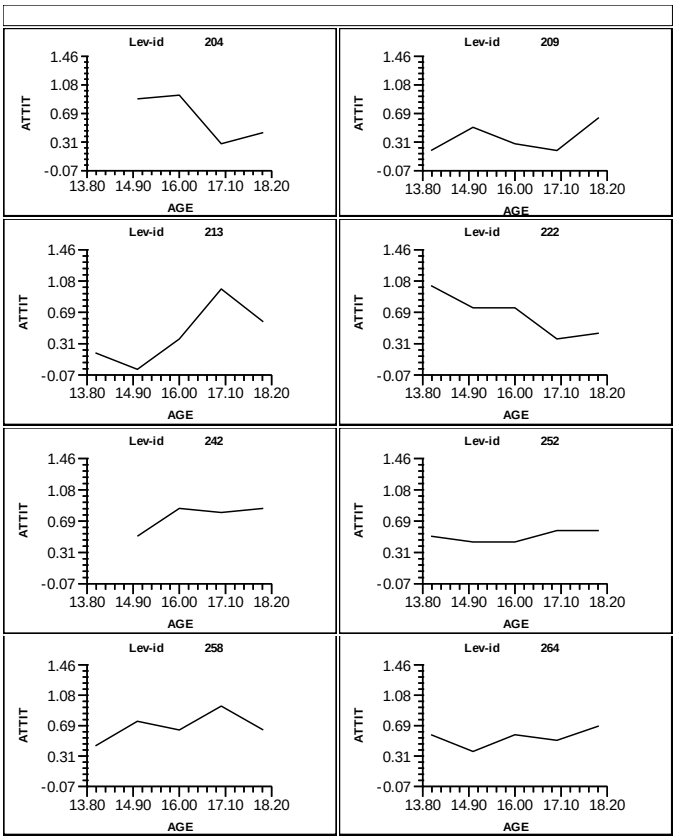
SC708: Hierarchical Linear Modeling
Instructor: Natasha Sarkisian
Class notes: Growth Curve Models

So far we've worked with one type of hierarchical data – students nested within schools. HLM can also be used to model longitudinal data where multiple observations over time are nested within one person.

We will use NYS2.MDM from Chapter 9 folder. This file contains data for a cohort of adolescents in the National Youth Survey, ages 14 to 18. The dependent variable ATTIT is a 9-item scale assessing attitudes favorable to deviant behavior (property damage, drug and alcohol use, stealing, etc.). The level-1 independent variables include: EXPO measuring exposure to deviant peers (students were asked how many of their friends engaged in the 9 deviant behaviors), AGE (age in years), AGES (age in years squared), AGE14 (age minus 14), AGE16 (age minus 16), AGE145 (age minus 14.5), and the three corresponding squared variables. Level 2 include person-level variables: FEMALE, MINORITY, INCOME, and an interaction term for MINFEM.

What we will study is how attitudes change over time, and what shapes that change. First, let's examine individual trajectories.





Now let's try to model these trajectories. First, we will assume that we can model them using a linear model. Therefore, we'll estimate an unconditional linear growth model:

```
Level-1 Model

       Y = B0 + B1*(AGE16) + R

Level-2 Model
       B0 = G00 + U0
       B1 = G10 + U1

 Sigma_squared =      0.02873

 Tau
 INTRCPT1,B0      0.04572      -0.00093
    AGE16,B1     -0.00093       0.00313


Tau (as correlations)
 INTRCPT1,B0  1.000 -0.078
    AGE16,B1 -0.078  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                          0.837
     AGE16, B1                          0.453
 ----------------------------------------------------

The outcome variable is    ATTIT
```

```
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                    Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
   INTRCPT2, G00           0.493325   0.014864    33.189       240    0.000
 For    AGE16 slope, B1
   INTRCPT2, G10           0.032357   0.005350     6.048       240    0.000
 ----------------------------------------------------------------------------

 The outcome variable is    ATTIT

 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                    Standard             Approx.
   Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
   INTRCPT2, G00           0.493325   0.014833    33.259       240    0.000
 For    AGE16 slope, B1
   INTRCPT2, G10           0.032357   0.005338     6.061       240    0.000
 ----------------------------------------------------------------------------

 Final estimation of variance components:
 ----------------------------------------------------------------------------
 Random Effect           Standard      Variance     df    Chi-square  P-value
                         Deviation     Component
 ----------------------------------------------------------------------------
 INTRCPT1,       U0        0.21383       0.04572   235    1754.38522    0.000
   AGE16 slope, U1         0.05595       0.00313   235     446.20764    0.000
  level-1,       R         0.16949       0.02873
 ----------------------------------------------------------------------------

Statistics for current covariance components model
 --------------------------------------------------
 Deviance                        = -99.676230
 Number of estimated parameters = 4
```

The mean growth trajectory is:
Attitude=.493 + .032*Age16

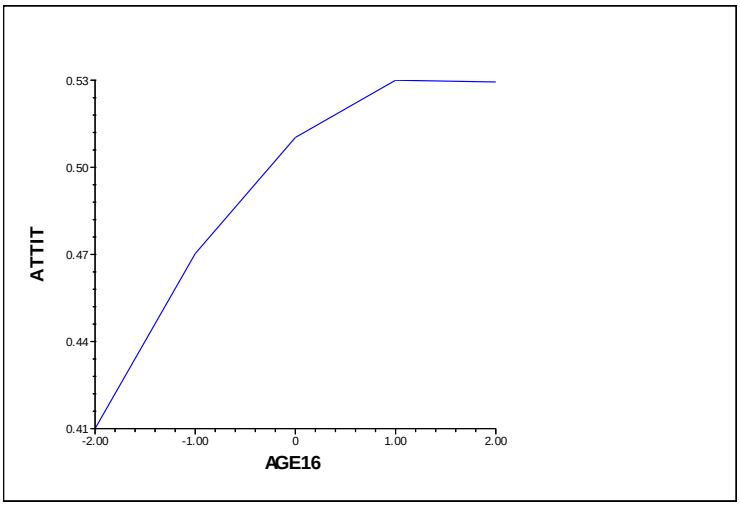
Now let's estimate an unconditional quadratic growth model and compare the fit:

```
Level-1 Model

       Y = B0 + B1*(AGE16) + B2*(AGE16S) + R

Level-2 Model
       B0 = G00 + U0
       B1 = G10 + U1
       B2 = G20 + U2

 Sigma_squared =      0.02291


Tau
 INTRCPT1,B0      0.05825      -0.00033      -0.00416
    AGE16,B1     -0.00033       0.00369      -0.00033
```

```
   AGE16S,B2     -0.00416      -0.00033       0.00118


Tau (as correlations)
 INTRCPT1,B0  1.000 -0.022 -0.502
    AGE16,B1 -0.022  1.000 -0.160
   AGE16S,B2 -0.502 -0.160  1.000

 ------------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ------------------------------------------------------
  INTRCPT1, B0                          0.822
     AGE16, B1                          0.530
    AGE16S, B2                          0.358
 ------------------------------------------------------

 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                    Standard            Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00           0.514018   0.017307    29.700       240    0.000
 For    AGE16 slope, B1
    INTRCPT2, G10           0.031463   0.005333     5.900       240    0.000
 For   AGE16S slope, B2
    INTRCPT2, G20          -0.010696   0.003652    -2.929       240    0.004
 ----------------------------------------------------------------------------

 Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                    Standard            Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00           0.514018   0.017270    29.764       240    0.000
 For    AGE16 slope, B1
    INTRCPT2, G10           0.031463   0.005320     5.914       240    0.000
 For   AGE16S slope, B2
    INTRCPT2, G20          -0.010696   0.003643    -2.936       240    0.004
 ----------------------------------------------------------------------------

 Final estimation of variance components:
 ----------------------------------------------------------------------------
 Random Effect           Standard      Variance     df    Chi-square  P-value
                         Deviation     Component
 ----------------------------------------------------------------------------
 INTRCPT1,       U0        0.24135       0.05825   222    1247.17000    0.000
    AGE16 slope, U1        0.06075       0.00369   222     503.78215    0.000
   AGE16S slope, U2        0.03437       0.00118   222     347.59593    0.000
  level-1,       R         0.15136       0.02291
 ----------------------------------------------------------------------------
 Statistics for current covariance components model
 --------------------------------------------------
 Deviance                        = -129.616127
 Number of estimated parameters = 7
```

The average growth trajectory becomes:
Attitude = 0.514+.031*Age16 – 0.011*Age16S

Let's graph it using model graphs in HLM; for that, we need to specify that AGE16S is a squared term for AGE16.



Our quadratic model does have smaller deviance value, but let's test the quadratic model against the linear model:

```
Variance-Covariance components test
 -----------------------------------
 Chi-square statistic          =     29.93990
 Number of degrees of freedom =    3
 P-value                       = 0.000
```

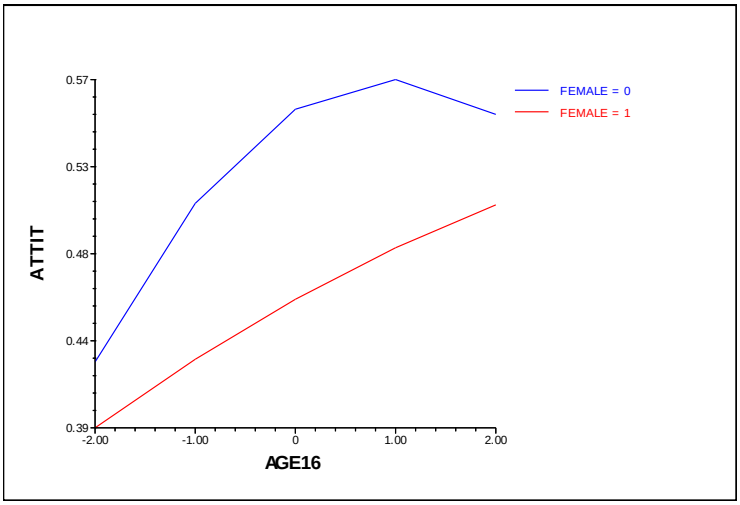
We conclude that quadratic model is a better fit, and proceed to estimating conditional models using person-level (time-invariant) predictors at first.

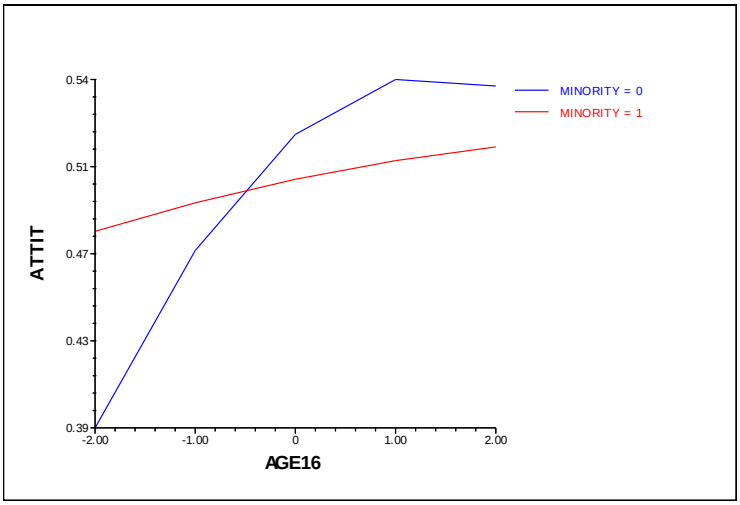
```
  The model specified for the fixed effects was:
 ----------------------------------------------------


   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ---------------
         INTRCPT1, B0      INTRCPT2, G00
                             FEMALE, G01
                           MINORITY, G02
$                            INCOME, G03
      AGE16 slope, B1      INTRCPT2, G10
                             FEMALE, G11
                           MINORITY, G12
$                            INCOME, G13
     AGE16S slope, B2      INTRCPT2, G20
                             FEMALE, G21
                           MINORITY, G22
$                            INCOME, G23

'$' - This level-2 predictor has been centered around its grand mean.


Level-1 Model

       Y = B0 + B1*(AGE16) + B2*(AGE16S) + R
```

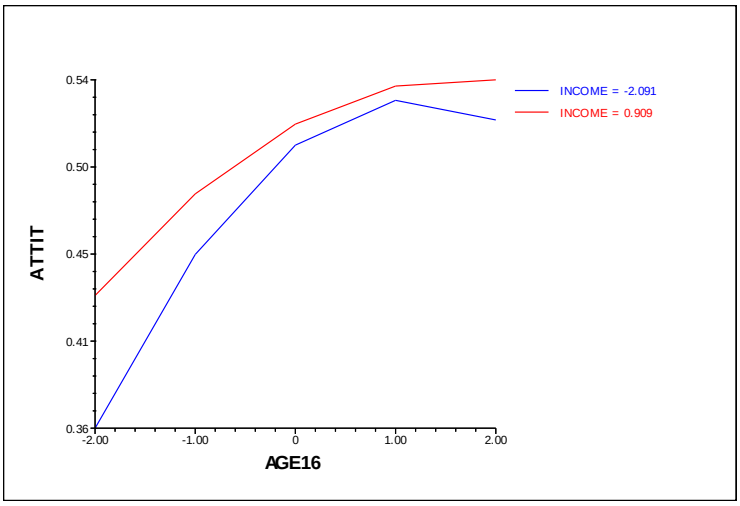
```
Level-2 Model
       B0 = G00 + G01*(FEMALE) + G02*(MINORITY) + G03*(INCOME) + U0
       B1 = G10 + G11*(FEMALE) + G12*(MINORITY) + G13*(INCOME) + U1
       B2 = G20 + G21*(FEMALE) + G22*(MINORITY) + G23*(INCOME) + U2

Sigma_squared =      0.02291

 Tau
 INTRCPT1,B0      0.05662      -0.00042      -0.00391
    AGE16,B1     -0.00042       0.00364      -0.00025
   AGE16S,B2     -0.00391      -0.00025       0.00112

Tau (as correlations)
 INTRCPT1,B0  1.000 -0.029 -0.492
    AGE16,B1 -0.029  1.000 -0.122
   AGE16S,B2 -0.492 -0.122  1.000

  Random level-1 coefficient   Reliability estimate
 ------------------------------------------------------
  INTRCPT1, B0                          0.818
     AGE16, B1                          0.527
    AGE16S, B2                          0.346
 ------------------------------------------------------
 Final estimation of fixed effects:
 ----------------------------------------------------------------------------
                                  Standard            Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00           0.562491   0.025856    21.754       237    0.000
      FEMALE, G01          -0.100283   0.034929    -2.871       237    0.005
    MINORITY, G02          -0.019852   0.044100    -0.450       237    0.653
      INCOME, G03           0.003602   0.007755     0.464       237    0.642
 For    AGE16 slope, B1
    INTRCPT2, G10           0.039149   0.008110     4.827       237    0.000
      FEMALE, G11          -0.003239   0.010823    -0.299       237    0.765
    MINORITY, G12          -0.028441   0.013824    -2.057       237    0.040
      INCOME, G13          -0.003963   0.002373    -1.670       237    0.096
 For   AGE16S slope, B2
    INTRCPT2, G20          -0.019852   0.005501    -3.609       237    0.001
      FEMALE, G21           0.014754   0.007364     2.003       237    0.046
    MINORITY, G22           0.012461   0.009468     1.316       237    0.190
      INCOME, G23           0.002798   0.001620     1.727       237    0.085
 ----------------------------------------------------------------------------

  Final estimation of fixed effects
 (with robust standard errors)
 ----------------------------------------------------------------------------
                                  Standard            Approx.
    Fixed Effect         Coefficient   Error      T-ratio   d.f.     P-value
 ----------------------------------------------------------------------------
 For       INTRCPT1, B0
    INTRCPT2, G00           0.562491   0.029658    18.966       237    0.000
      FEMALE, G01          -0.100283   0.034379    -2.917       237    0.004
    MINORITY, G02          -0.019852   0.039082    -0.508       237    0.611
      INCOME, G03           0.003602   0.006930     0.520       237    0.603
 For    AGE16 slope, B1
    INTRCPT2, G10           0.039149   0.007686     5.094       237    0.000
      FEMALE, G11          -0.003239   0.010304    -0.314       237    0.753
    MINORITY, G12          -0.028441   0.014088    -2.019       237    0.044
      INCOME, G13          -0.003963   0.002075    -1.910       237    0.057
 For   AGE16S slope, B2
```

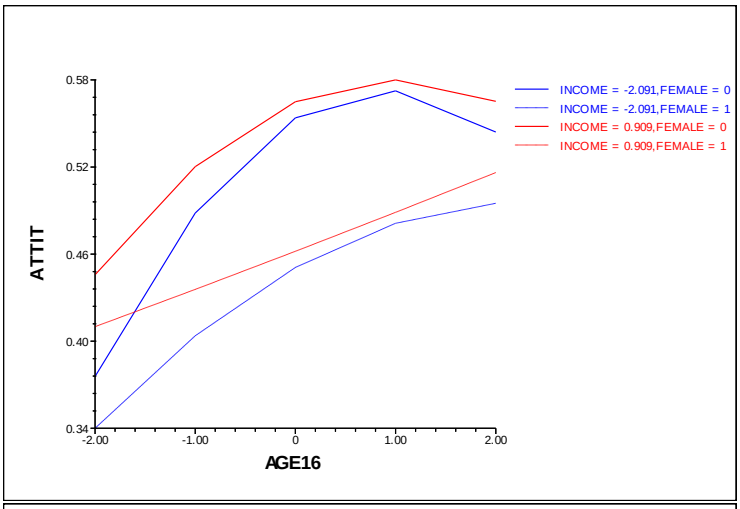
```
   INTRCPT2, G20          -0.019852    0.006129    -3.239       237    0.002
     FEMALE, G21           0.014754    0.007121     2.072       237    0.039
   MINORITY, G22           0.012461    0.009555     1.304       237    0.194
     INCOME, G23           0.002798    0.001383     2.023       237    0.044
-----------------------------------------------------------------------------

Final estimation of variance components:
-----------------------------------------------------------------------------
Random Effect             Standard      Variance     df    Chi-square  P-value
                          Deviation     Component
-----------------------------------------------------------------------------
INTRCPT1,       U0          0.23795       0.05662   219    1196.00045    0.000
   AGE16 slope, U1          0.06030       0.00364   219     495.23926    0.000
  AGE16S slope, U2          0.03342       0.00112   219     336.68827    0.000
 level-1,       R          0.15135       0.02291
-----------------------------------------------------------------------------
```

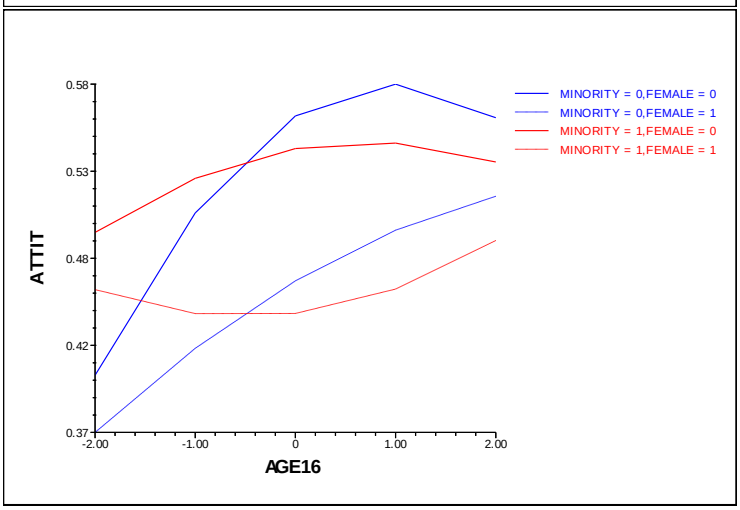
Let's explore this model graphically:











Finally, let's estimate a quadratic growth model with a time-varying covariate (EXPO). Here, we will use EXPO grand-centered. If we wanted to take this analysis one step further, we could have created a mean exposure variable on person level (level 2) and then used EXPO group centered on level 1 and mean of EXPO on level 2.

```
   Level-1                  Level-2
   Coefficients             Predictors
 ----------------------   ---------------
         INTRCPT1, B0      INTRCPT2, G00
                             FEMALE, G01
                           MINORITY, G02
$                            INCOME, G03
 %     EXPO slope, B1      INTRCPT2, G10
                             FEMALE, G11
                           MINORITY, G12
$                            INCOME, G13
      AGE16 slope, B2      INTRCPT2, G20
                             FEMALE, G21
                           MINORITY, G22
$                            INCOME, G23
     AGE16S slope, B3      INTRCPT2, G30
                             FEMALE, G31
                           MINORITY, G32
$                            INCOME, G33

'%' - This level-1 predictor has been centered around its grand mean.
'$' - This level-2 predictor has been centered around its grand mean.


Level-1 Model

       Y = B0 + B1*(EXPO) + B2*(AGE16) + B3*(AGE16S) + R

Level-2 Model
       B0 = G00 + G01*(FEMALE) + G02*(MINORITY) + G03*(INCOME) + U0
       B1 = G10 + G11*(FEMALE) + G12*(MINORITY) + G13*(INCOME) + U1
       B2 = G20 + G21*(FEMALE) + G22*(MINORITY) + G23*(INCOME) + U2
       B3 = G30 + G31*(FEMALE) + G32*(MINORITY) + G33*(INCOME) + U3

 Sigma_squared =      0.02030

 Tau
 INTRCPT1,B0      0.02273      -0.00288       0.00068      -0.00147
     EXPO,B1     -0.00288       0.03327      -0.00273       0.00185
    AGE16,B2      0.00068      -0.00273       0.00276      -0.00034
   AGE16S,B3     -0.00147       0.00185      -0.00034       0.00058


Tau (as correlations)
 INTRCPT1,B0  1.000 -0.105  0.086 -0.405
     EXPO,B1 -0.105  1.000 -0.285  0.420
    AGE16,B2  0.086 -0.285  1.000 -0.270
   AGE16S,B3 -0.405  0.420 -0.270  1.000

 ----------------------------------------------------
  Random level-1 coefficient   Reliability estimate
 ----------------------------------------------------
  INTRCPT1, B0                         0.342
      EXPO, B1                         0.062
     AGE16, B2                         0.330
    AGE16S, B3                         0.166
 ----------------------------------------------------

 Final estimation of fixed effects:
 ------------------------------------------------------------------------------
                                      Standard            Approx.
```

Fixed Effect	Coefficient	Error	T-ratio	d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.536548	0.018864	28.443	237	0.000
FEMALE, G01	-0.087195	0.025319	-3.444	237	0.001
MINORITY, G02	-0.003917	0.032033	-0.122	237	0.903
INCOME, G03	0.006434	0.005601	1.149	237	0.252
For EXPO slope, B1					
INTRCPT2, G10	0.551921	0.041454	13.314	237	0.000
FEMALE, G11	-0.048549	0.058298	-0.833	237	0.406
MINORITY, G12	-0.404139	0.071710	-5.636	237	0.000
INCOME, G13	-0.042315	0.013089	-3.233	237	0.002
For AGE16 slope, B2					
INTRCPT2, G20	0.018852	0.007483	2.519	237	0.013
FEMALE, G21	0.008663	0.009922	0.873	237	0.384
MINORITY, G22	-0.008015	0.012682	-0.632	237	0.528
INCOME, G23	-0.001653	0.002179	-0.759	237	0.449
For AGE16S slope, B3					
INTRCPT2, G30	-0.011305	0.004845	-2.333	237	0.021
FEMALE, G31	0.014522	0.006476	2.242	237	0.026
MINORITY, G32	0.003959	0.008345	0.474	237	0.635
INCOME, G33	0.002238	0.001416	1.580	237	0.115

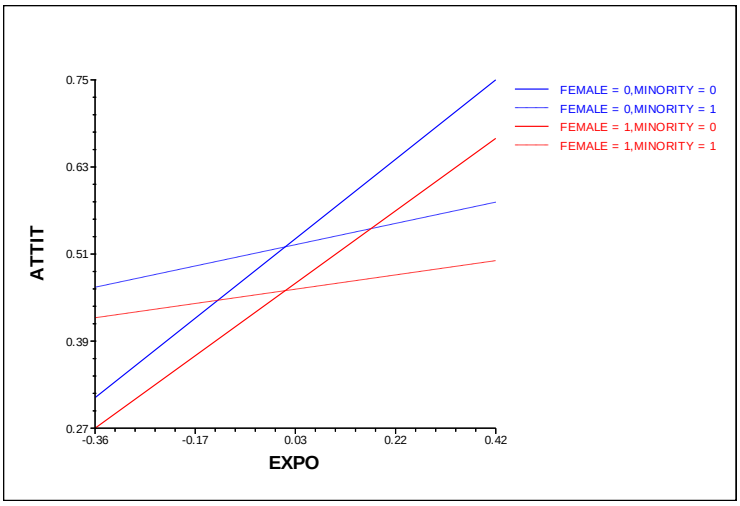
Final estimation of fixed effects
(with robust standard errors)

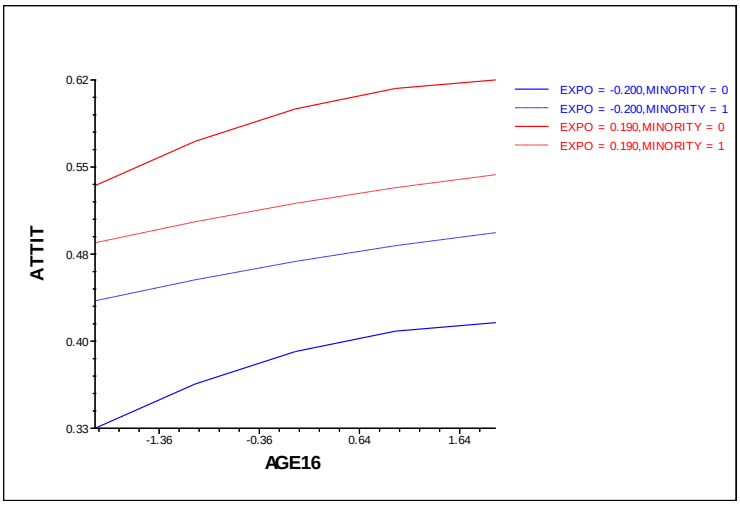
Fixed Effect	Coefficient	Standard Error	T-ratio	Approx. d.f.	P-value
For INTRCPT1, B0					
INTRCPT2, G00	0.536548	0.019993	26.837	237	0.000
FEMALE, G01	-0.087195	0.024535	-3.554	237	0.001
MINORITY, G02	-0.003917	0.032658	-0.120	237	0.905
INCOME, G03	0.006434	0.005623	1.144	237	0.254
For EXPO slope, B1					
INTRCPT2, G10	0.551921	0.038407	14.370	237	0.000
FEMALE, G11	-0.048549	0.057455	-0.845	237	0.399
MINORITY, G12	-0.404139	0.072271	-5.592	237	0.000
INCOME, G13	-0.042315	0.014359	-2.947	237	0.004
For AGE16 slope, B2					
INTRCPT2, G20	0.018852	0.007315	2.577	237	0.011
FEMALE, G21	0.008663	0.009423	0.919	237	0.359
MINORITY, G22	-0.008015	0.013110	-0.611	237	0.541
INCOME, G23	-0.001653	0.002002	-0.826	237	0.410
For AGE16S slope, B3					
INTRCPT2, G30	-0.011305	0.004920	-2.298	237	0.022
FEMALE, G31	0.014522	0.006166	2.355	237	0.019
MINORITY, G32	0.003959	0.008755	0.452	237	0.651
INCOME, G33	0.002238	0.001269	1.763	237	0.079

Final estimation of variance components:

Random Effect		Standard Deviation	Variance Component	df	Chi-square	P-value
INTRCPT1,	U0	0.15078	0.02273	197	402.26089	0.000
EXPO slope,	U1	0.18240	0.03327	197	244.37700	0.012
AGE16 slope,	U2	0.05252	0.00276	197	307.13303	0.000
AGE16S slope,	U3	0.02415	0.00058	197	250.50418	0.006
level-1,	R	0.14248	0.02030			

Some graphs to illustrate these findings:


0.75
0.63
0.51
0.39
0.27
ATTIT
-0.36
-0.17
0.03
0.22
0.42
EXPO
FEMALE = 0,MINORITY = 0
FEMALE = 0,MINORITY = 1
FEMALE = 1,MINORITY = 0
FEMALE = 1,MINORITY = 1


0.62
0.55
0.48
0.40
0.33
ATTIT
-1.36
-0.36
0.64
1.64
AGE16
EXPO = -0.200,MINORITY = 0
EXPO = -0.200,MINORITY = 1
EXPO = 0.190,MINORITY = 0
EXPO = 0.190,MINORITY = 1