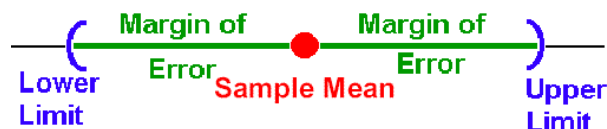


Handout: Margin of Error in Polls

Margin of error = the "radius" (or half the width) of a confidence interval

***Margin of error and confidence interval for a proportion***

For opinion polls, we usually estimate confidence intervals not for means but for proportions. The formula is the same as for the mean but we do not need a separate estimate of standard deviation; the variance of a proportion is estimated as $\hat{p}(1 - \hat{p})$ where $\hat{p}$ is our sample proportion itself, and the standard deviation is the square root of that. So the confidence interval for proportion is:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Example:

For a sample size $n = 900$ and an estimated sample proportion = 0.2:

$$95\% \text{ CI} = 0.2 \pm 1.96 \cdot \sqrt{0.2 \cdot 0.8 / 900} = 0.2 \pm 0.026$$

Formal statement: Probability($.174 \leq p \leq 0.226$) = .95

We are 95% confident that the true proportion in the population is between .174 and .226.

$$\text{Margin of error} = 0.026 \cdot 100 = \pm 2.6\%$$

Maximum margin of error in a poll

Opinion polls usually report the maximum margin of error for any percentage from that poll. The margin of error for a particular percentage is usually smaller than that value. The maximum margin of error always occurs for percentage=50% (which is proportion = .5); that's because when the sample is split 50-50, there is the most uncertainty in the outcome. So based on the sample size, the maximum margin of error is calculated as:

$$\begin{aligned} \text{CI}(p) &= \pm 1.96 \times \sqrt{\frac{.5 \times .5}{n - 1}} \\ &\approx \pm 2 \times \frac{.5}{\sqrt{n}} \\ &\approx \pm \frac{1}{\sqrt{n}} \end{aligned}$$

Example:

Sample size $n = 400$

$$\text{Maximum margin of error} = 1/\sqrt{n} = 1/\sqrt{400} = 1/20 = 0.05$$

Answer: maximum margin of error = $\pm 5\%$

Important: margin of error applies only when we consider the whole sample, not particular subgroups (e.g., women, Republicans, etc.)